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PROPAGATION OF QUASIACOUSTIC PULSES IN AN ELASTOPLASTIC MEDIUM

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Expressions for the velocity of a plastic shock wave and phase velocity of longitudinal waves in an elastoplastic medium with hardening are obtained in a quasiacoustic approximation. An analytical solution of the problem of shock-pulse attenuation is constructed. A special feature of the amplitude of the attenuating plastic shock wave is that it reaches the amplitude of the elastic precursor in a finite time, whereas in hydrodynamics, the amplitude of a quasiacoustic shock pulse tends to zero asymptotically.

Key words: *elastoplastic medium, isotropic hardening, nonlinear waves, shock wave.*

Introduction. Studying the evolution of nonlinear waves generated by explosive or shock loading of materials and structures is of scientific and practical interest. These studies are usually performed under conditions of pulsed loading with pressure amplitudes varying from several pascals to tens of gigapascals [1]. In this case, one can usually assume that the waves generated are weak in the sense of the small ratio of pressure to the bulk modulus of the material and use approximate relations of nonlinear acoustics [2, 3] to model shock-wave processes.

In the present paper, the propagation of nonlinear acoustic waves in an elastoplastic medium is studied using the model proposed in [4].

1. Stationary Shock Waves. We consider propagation of plane longitudinal waves under uniaxial strain. In this case, the quantities governing the propagation of these waves are functions of a single variable $X - D_{rm}t$ ($X = x_1|_{t=0}$ and D_{rm} is the Lagrange phase velocity of wave propagation). Let a compression wave bring the medium from the state $(\rho, u_1, \sigma_1, \tau)_r$ as $t \to -\infty$ to the state $(\rho, u_1, \sigma_1, \tau)_m$ as $t \to \infty$. Here $\tau = -(\sigma_1 - \sigma_2)/2$, σ_i are the principal stresses, u_1 is the velocity of the medium, and ρ is the density. The laws of conservation imply the relations (internal-friction viscosity and heat conduction are ignored)

$$u_1 - (u_1)_r = \rho_0 D_{rm}(1/\rho_r - 1/\rho), \quad \sigma_1 - (\sigma_1)_r = -(\rho_0 D_{rm})^2(1/\rho_r - 1/\rho), \qquad (1)$$
$$E - E_r + (\sigma_1 + (\sigma_1)_r)(1/\rho_r - 1/\rho)/2 = 0,$$

where E is the internal energy. Equations (1) describe both smooth and discontinuous jumplike variations of quantities in the wave. It is well known that the internal energy of an isotropic medium is a function of the strain-tensor invariants and entropy S. For convenience, this dependence can be written as $E = E(\rho, D, \Delta, S)$, where D and Δ are the invariants of the deviator of the effective elastic-strain tensor [5].

We introduce the small parameters ε (ratio of the stress amplitude to the bulk modulus) and $\nu = (C_{\rm long}^2 - C_0^2)/(2C_0^2) = 2G/(3\rho_0 C_0^2)$ (G is the shear modulus, $C_{\rm long}$ is the phase velocity of the longitudinal elastic waves, and C_0 is the volume velocity of sound). Thus, the stress-deviator components are assumed to be quantities of a higher order of smallness compared to the average stress [dimensionless average stress is a quantity of order $O(\varepsilon)$]. We expand the internal energy E into a power series of the increments in density $\rho' = (\rho - \rho_0)/\rho_0$ and entropy $S' = T_0(S - S_0)/C_0^2$ and invariants D and Δ with allowance for $\partial E/\partial D\Big|_0 = 2G/\rho_0$ and retain terms up to the second order of smallness in the hydrodynamic part of the stress tensor. As a result, we obtain

$$\sigma_1' = \sigma_1/(\rho_0 C_0^2) = -[\rho' + \alpha\rho'^2/2 + \Gamma S' + O(\varepsilon^3 + \varepsilon^2\nu)] - 3\nu(\psi + O(\varepsilon^2)). \qquad (2)$$

Here $\psi = 2\tau/(3G)$, the parameter $\alpha = 4 + \rho_0^3 E_{\rho\rho\rho}\Big|_S/C_0^2$ is determined from the equation of state and can be calculated, for example, in terms of the adiabatic derivative of the bulk modulus with respect to pressure, determined

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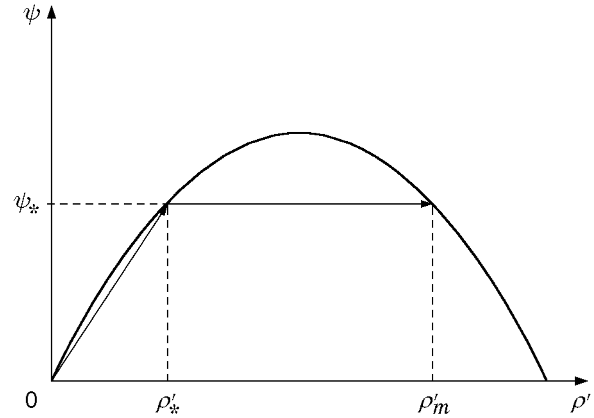

Fig. 1

from the available experimental data of [6], T_0 is the initial temperature, and Γ is the Grüneisen coefficient. The amplitude ψ depends on the quantity $Y/(3G)$ (Y is the tensile yield point). For metals, we have $Y/(3G) \sim 10^{-3}$; therefore, despite the fact that the introduced small parameter ν is rather "large" for metals ($\nu \leqslant 0.3$), the term $3\nu\psi$ in (2) is small compared to the leading term of the expansion $|\rho'| \sim \varepsilon$. For stresses that occur in typical shock-wave tests, we have $\varepsilon \sim 0.1$.

System (1), (2) implies the relations for the entropy increment, velocity D_{rm}, and dependence $\psi(\rho')$:

$$[S'] = (\alpha+2)[\rho']^3/12 + (9/4)\nu(\psi_m + \psi_r)[2\rho'/3 - \psi] + O(\varepsilon^4 + \varepsilon^3\nu); \tag{3}$$

$$(D_{rm}/C_0)^2 = 1 + (\alpha+2)(\rho'_r + \rho'_m)/2 + 3\nu[\psi]/[\rho'] + O(\varepsilon^2 + \varepsilon\nu); \tag{4}$$

$$3\nu(\psi - \psi_r)/2 = \delta(\rho' - \rho'_r) - (\alpha+2)(\rho'^2 - \rho'^2_r)/4 + O(\varepsilon^2 + \varepsilon\nu), \tag{5}$$

where $[\,\cdot\,] = (\,\cdot\,)_m - (\,\cdot\,)_r$ and $\delta = (D^2_{rm}/C^2_0 - 1)/2$. The first and second terms on the right side of Eq. (3) determine the increments in entropy due to variation in density in the shock wave and the work done in plastic strain, respectively. Relation (5) describes possible states inside the wave and determines the dependence of the shear stress $(9/4)\nu(\psi - \psi_r)$ on the true strain ρ'. To obtain stationary solutions, one should combine relation (5) with a constitutive equation relating ψ and ρ'.

For small strains, the quantity ψ is related to the plastic strain by the formula

$$\dot\varepsilon^p_1 = -2\dot\rho'/3 + \dot\psi \tag{6}$$

(the dot denotes the derivative with respect to time).

We consider a model of an elastoplastic medium of the Prandtl–Reuss type and Mises yield criterion for the case of uniaxial strain. Using the notation introduced, we write the governing equations as

$$\dot\psi = 2\dot\rho/3 \quad \text{for} \quad |\psi| \leqslant \psi_*, \qquad \psi = \psi_* \operatorname{sign}\psi \quad \text{for} \quad |\psi| > \psi_*. \tag{7}$$

Here $\psi_* = Y/(3G)$. We first consider the case of a constant yield point ($Y = \text{const}$). An elementary analysis of Eqs. (5) and (7) gives the following results. For $\rho'_r = \psi_r = 0$ (the wave propagates over an undisturbed medium), there can exist a unique stationary solution for $\rho'_m > \rho'_*$ ($\rho'_* = 3\psi_*/2$) in the form of a sequence of two strong discontinuities (shock waves). In the elastic shock wave, the medium is brought in a jumplike manner from the state (0,0) to the state (ρ'_*, ψ_*); in the plastic shock wave, it is brought from the state (ρ'_*, ψ_*) to the state $(4\nu/(\alpha+2), \psi_*)$, i.e., the amplitude of the stationary wave is given by $\rho'_m = 4\nu/(\alpha+2)$ and $\psi_m = \psi_*$. In this case, $\delta = (\alpha+2)\rho'_*/4 + \nu$. Figure 1 shows the dependence $\psi(\rho')$ [formula (5)]. The phase shift between the jumps is indeterminate (it may be arbitrary); only the sequence of the jumps is determined. If $\rho'_m > 4\nu/(\alpha+2)$, the plastic shock wave overtakes the elastic shock wave; otherwise, it lags behind the elastic shock wave.

We further use the model of isotropic hardening in which the yield point depends on the scalar parameter Λ. We write the flow law in the form

$$d\varepsilon^p_{ij} = (3/4)s_{ij}\,d\Lambda/\sqrt{(3/8)s_{kl}s_{kl}}. \tag{8}$$

Here ε^p_{ij} are the components of the plastic-strain tensor and s_{ij} are the stress-deviator components. It follows from (8) that the parameter Λ is related to the hardening parameter W^p, which is equal to the work done in

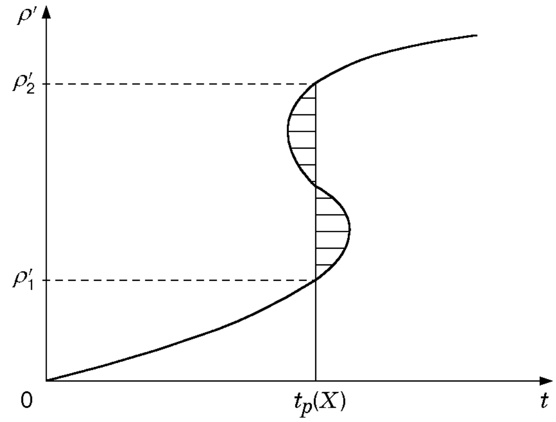

Fig. 2

plastic strains, by the transformation $dW^p = Y(\Lambda)\,d\Lambda$. Integration and substitution of $W^p = \varphi(\Lambda)$ into the known dependence $Y_s(W^p)$ yields $Y(\Lambda) = Y_s(\varphi(\Lambda))$. A similar transformation was used in [7]. Using (6) and (8), we obtain $\dot{\Lambda} = 2\dot{\rho}'/3 - \dot{Y}/(3G)$ for the plastic wave. Integrating this equation with the initial conditions $\Lambda = 0$, $\rho' = \rho'_*$, and $Y(0) = Y_0$ and taking into account the relation $\rho'_* = Y_0/(2G)$, we obtain

$$\Lambda = 2\rho'/3 - Y/(3G). \tag{9}$$

Thus, given the relation $Y = Y(\Lambda)$, one can obtain a stationary solution for the elastoplastic wave by setting $\rho_r = \rho_*$, $\psi_r = \psi_*$ and $\delta = (\alpha+2)\rho'_*/4+\nu$ in Eq. (5) and solving it simultaneously with Eq. (9). In particular, from (5) there follows the relation $Y_m - Y_0 = 2G(\rho'_m - \rho'_*)(1 - (\alpha+2)\rho'_m/(4\nu))$, which shows that stationary waves with an amplitude $\rho'_* < \rho'_m < 4\nu/(\alpha+2)$ can occur in a hardening medium $Y_m > Y_0$. In the case of $Y(\Lambda) = Y_0 + Y_1\Lambda$ (Y_0 and Y_1 are constants), for $\rho' > \rho'_*$, one can construct a solution in the form of a plastic shock wave that brings the medium from the state (ρ'_*, ψ_*) to the state with parameters $\rho'_m = 4\nu(1-k)/(\alpha+2)$ and $\psi_m = \psi_* + k(2\rho'_m/3 - \psi_*)$, where $k = (1+3G/Y_1)^{-1}$, $0 < k < 1 - (\alpha+2)\rho'_*/(4\nu)$, $\rho'_* = Y_0/(2G)$, and $\psi_* = Y_0/(3G)$.

Relation (4) is valid for jumplike variations of quantities in any longitudinal wave (in this case, the subscripts m and r refer quantities immediately ahead of and behind the jump) for any elastoplastic medium, since the constitutive equation was not used in deriving relation (4). Discontinuous solutions in the form of shock waves exist owing to the nonunique profile of the wave formed as a result of tumbling of the initially smooth solution, where the Lagrange propagation velocity of fixed strain levels $C(\rho')$ increases with ρ'. In this approximation, we obtain

$$C^2 = \Big(\frac{dX}{dt}\Big)^2\Big|_{\rho'} = C_0^2\Big(1 + (\alpha+2)\rho' + 3\nu\Big(\frac{\partial\psi}{\partial\rho'}\Big)_X\Big) + O(\varepsilon^2 + \varepsilon\nu). \tag{10}$$

It should be noted that expression (10) can be obtained from (4) if $[\rho']$ and $[\psi]$ tend to zero simultaneously and $\rho'_r \to \rho'$ and $\rho'_m \to \rho'$. In this case, one can construct a discontinuous solution and determine the front location using the so-called equal-area rule [2, 3]. We show that the velocity of the shock-wave front determined by formula (4) satisfies this rule. Let the discontinuity occupy the position $t = t_p(X)$ at a given moment (Fig. 2). The shaded area in Fig. 2 is equal to the integral

$$J = \int_{\rho'_1}^{\rho'_2} (t(\rho', X) - t_p(X))\,d\rho'.$$

Here $t(\rho', X)$ is the dependence determined by relation (10). In Fig. 2, the shaded areas on the left and on the right of the straight line $t = t_p(X)$ are equal. Differentiating the expression for J with respect to X, with allowance for (4) and (10), we obtain

$$\frac{\partial}{\partial X}J = \int_{\rho'_1}^{\rho'_2} \Big(\frac{1}{C(\rho')} - \frac{1}{D_{12}}\Big)\,d\rho' = O(\varepsilon^3 + \varepsilon^2\nu).$$

Obviously, the shaded area vanishes at the point where the discontinuity X_p occurs. Hence, by virtue of the calculations performed, the equal-area rule is satisfied asymptotically for $X > X_p$, at least at distances $X - X_p \leqslant O(\varepsilon^{-1})$.

Let us determine the velocities of shock waves and the velocities $C(\rho')$ for different constitutive equations. For elastic strain, from (4), (6), and (10) we obtain

$$D_{rm}^2 = C_{\text{long}}^2 + C_0^2(\alpha+2)(\rho'_r + \rho'_m)/2, \qquad C^2 = C_{\text{long}}^2 + C_0^2(\alpha+2)\rho', \tag{11}$$

where C_{long} is the phase velocity of longitudinal elastic waves in an undisturbed medium. For the plastic discontinuity in the Prandtl–Reuss medium with the Mises yield criterion, from (6) and (8) we obtain

$$[\Lambda] = 2[\rho']/3 - [Y]/(3G), \qquad [\Lambda] \geqslant 0. \tag{12}$$

Substituting (12) into (4) and (10), we obtain expressions for the velocity of the plastic shock wave and Lagrange propagation velocity of perturbations for active loading ($\dot{\rho}' > 0$), which have the following form in the model of a hardening medium with the plasticity criterion $Y(\Lambda)$:

$$\rho_0 D_{rm}^2 = \rho_0 C_0^2\Big(1 + \frac{\alpha+2}{2}(\rho'_r + \rho'_m)\Big) + \frac{4}{3}\,\frac{[Y]}{3[\Lambda] + [Y]/G},$$
$$\rho_0 C^2 = \rho_0 C_0^2(1 + (\alpha+2)\rho') + \frac{4}{3}\,\frac{dY}{d\Lambda}\Big/\Big(3 + \frac{1}{G}\,\frac{dY}{d\Lambda}\Big). \tag{13}$$

Thus, the shock-wave velocity is uniquely determined by the known state ahead of the front and the value of Λ_m ($\Lambda_m > \Lambda_r$) behind the front, using Eq. (12) and the specified relation $Y(\Lambda)$. For small strains, the first formula in (13) is identical to the formula of [8] if Y and Λ are expressed in terms of the shear yield point and the hardening function of [8], respectively. For the ideal-plasticity model, the condition $[Y] = 0$ holds, and from (13) we obtain

$$D_{rm}^2 = C_0^2(1 + (\alpha+2)(\rho'_r + \rho'_m)/2), \quad C^2 = C_0^2(1 + (\alpha+2)\rho'). \tag{14}$$

In this case, the formulas for velocities are similar to those used in nonlinear acoustics [2, 3]. The shock waves should satisfy the stability condition [9]

$$C(\rho'_r) < D_{rm} < C(\rho'_m). \tag{15}$$

This condition has a simple physical meaning: the shock wave is subsonic with respect to particles immediately behind the front and supersonic with respect to particles immediately ahead of the front. Condition (15) is valid for shock waves in an elastic material (11), ideal plastic material (14) and, particularly, material with hardening (13) where $[Y(\Lambda)] = Y_1[\Lambda]$.

2. Attenuation of a Shock Pulse in an Elastoplastic Medium with Hardening. Stress pulses produced by thin impactors, short laser pulses, and detonation of layers of condensed explosives have a distinct front and a region of gentle decrease. We consider the following problem. Let the pressure monotonically decreasing with time be suddenly applied to the boundary $z = 0$ and $z = X/(C_0 t_0)$:

$$\frac{1}{2}\,V(0,\xi) = \begin{cases} 0, & -\infty < \xi < 0, \\ F(\xi)\ \ (F(\xi) > 0,\ dF(\xi)/d\xi < 0), & 0 \leqslant \xi < +\infty. \end{cases} \tag{16}$$

Here $V = -2\sigma'_1$. We denote the current amplitude of the wave by $V_m(z)$. On the boundary $z = 0$, we have $F(0) = V_m(0) = V_{m0}$.

To describe the propagation of a shock pulse in a half-space, we use the model equation proposed in [4]:

$$\frac{\partial V}{\partial z'} - \frac{1}{2}\,V\,\frac{\partial V}{\partial \xi} - 3\nu'\,\frac{\partial \psi}{\partial \xi} = 0. \tag{17}$$

Here $\xi = t' - z$, $z' = z(\alpha+2)/2$, $\nu' = 2\nu/(\alpha+2)$, and $t' = t/t_0$. Below, the primes at z and ν are omitted.

We use the constitutive equations of the deformation theory of plasticity and the model of linear hardening according to the Prandtl scheme in which the Bauschinger effect is ignored and the tensile and compressive yield points are assumed to be equal. For uniaxial deformation, the constitutive equations in the variables V and ψ have the form (Fig. 3)

$$\frac{\partial \psi}{\partial \xi} = \frac{1}{3}\,\frac{\partial V}{\partial \xi} \tag{18}$$

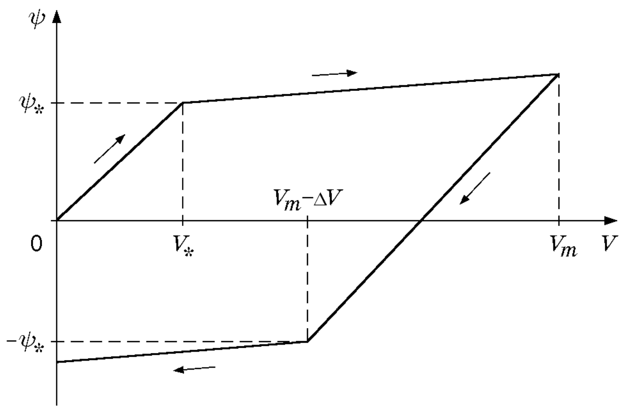

Fig. 3

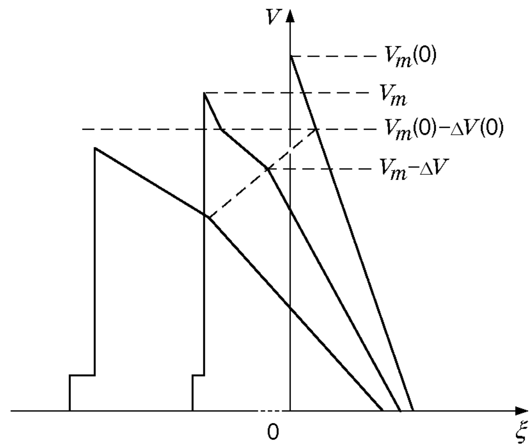

Fig. 4

for $\psi < \psi_* = Y_0/(3G)$ and for $V_m > V > V_m - \Delta V$ in the case of unloading, and

$$\frac{\partial \psi}{\partial \xi} = \frac{1}{3}\, k\, \frac{\partial V}{\partial \xi} \tag{19}$$

for $V_* \leqslant V \leqslant V_m$ in the case of loading and for $V_m - \Delta V \geqslant V$ in the case of unloading. In (18) and (19), Y_0 is the yield point, and $k = G'/G$ (G' is the modulus of linear hardening of the material). It is obvious that

$$V_m - \Delta V = V_m(1-k) - V_*(2-k). \tag{20}$$

If $V_{m0} \leqslant 4\nu(1-k)$, for $z > 0$, the strong-discontinuity wave is split into an elastic shock wave with an amplitude V_* (elastic precursor) and a plastic shock wave. The velocity of the latter is calculated by the formula similar to that for calculating the velocity in an elastic medium (see Sec. 1):

$$\frac{d\xi_p}{dz} = -\Big[\frac{1}{4}\,(V_m + V_*) + \nu k\Big] \tag{21}$$

(ξ_p is the coordinate of the front of the plastic shock wave). We further assume that the relation $V_{m0} \leqslant 4\nu(1-k)$ is valid.

For the unloading region, from (17)–(19) we obtain

$$\frac{dV}{dz} = 0 \quad \text{for} \quad \frac{d\xi}{dz} = \begin{cases} -(V/2+\nu), & V_m \geqslant V > V_m - \Delta V, \\ -(V/2+\nu k), & V_m - \Delta V \geqslant V > 0. \end{cases} \tag{22}$$

We consider the interval of amplitude variation $V_m(0) \geqslant V_m \geqslant V_m(0) - \Delta V(0)$ (Fig. 4). It follows from (20) that $V_m(0) - \Delta V(0) = V_{m0}(1-k) - V_*(2-k)$. In accordance with (22), the amplitude attenuation in this interval is affected only by the elastic part of the profile specified on the boundary. Let $\tilde{\xi}$ be the coordinate of a point on the profile (16), such that the value of $V = 2F(\tilde{\xi})$ lies within this interval: $V_m(0) \geqslant V \geqslant V_m(0) - \Delta V(0)$. Relation (22) shows that, at subsequent moments $z > 0$, this point has the coordinate $\xi = F^{-1}(V/2) - (V/2+\nu)z$ (F^{-1} is a function inverse to F). Substituting the values of $\xi = \xi_p$ and $V = V_m$ that correspond to the moment of intersection of the characteristic with the shock-wave front into this relation and combining the resulting expression with (21), after some transformations, we obtain the equation for $V_m(z)$

$$\frac{dV_m}{dz} = -\frac{1}{2}\,\frac{V_m - V_* + 4\nu(1-k)}{z - (F^{-1}(V_m/2))'}. \tag{23}$$

Here the prime denotes the derivative of F^{-1} with respect to the argument. This equation is supplemented by the initial condition $z = 0$ and $V_m(0) = V_{m0}$. The solution is constructed up to $z = z_1$ [z_1 is determined from relation (20): $V_m(z_1) = V_{m0}(1-k) - V_*(2-k)$]. We assume that the solution of Eq. (23) is known and denote it by $V_m = f_1(z)$.

Using (16) and (22), we construct the solution of the equation for the unloading region in the interval $0 < z \leqslant z_1$ (Fig. 4):

$$\frac{1}{2}V = \begin{cases} F(\xi + (V/2+\nu)z), & V_m(z) \geqslant V \geqslant V_m(0) - \Delta V(0), \\ F(\xi + (V/2+\nu)z - (1-k)\nu z_{pe}(V)), & V_m(0) - \Delta V(0) > V > V_m(z) - \Delta V(z), \\ F(\xi + (V/2+\nu k)z), & V_m(z) - \Delta V(z) \geqslant V > 0. \end{cases} \tag{24}$$

Here $z_{pe}(V)$ is the moment the elastic-unloading wave arrives at the point V, which lied initially in the plastic region. The second equality follows from the relation

$$\xi = F^{-1}(V/2) - (V/2+\nu k)z_{pe} - (V/2+\nu)(z - z_{pe}) \tag{25}$$

implied by (22). To calculate $z_{pe}(V)$, it is necessary to consider the interval $V_m(z_1) > V > V_m(z_1) - \Delta V(z_1)$. The point V from this interval lies initially in the plastic region and then in the elastic region of the flow. The moment $z_{pe}(V)$ when the point crosses the boundary between these regions is determined from Eq. (20): $f_1(z_{pe})(1-k) - V_*(2-k) = V$, which yields

$$z_{pe} = f_1^{-1}((V + V_*(2-k))/(1-k)) \tag{26}$$

(f_1^{-1} is a function inverse to f_1). Thus, the known solution for the amplitude $V_m = f_1(z)$ with (20) and (26) determines completely the unloading wave (24) in the interval $0 < z \leqslant z_1$.

We consider the interval $V_m(z_1) > V_m > V_m(z_1) - \Delta V(z_1)$, where the amplitude varies (decays). Relations (25) and (26) for $\xi = \xi_p$ and $V = V_m$ combined with Eq. (21) yield the equation for the shock-wave amplitude in this interval:

$$\frac{dV_m}{dz} = -\frac{1}{2}\,\frac{V_m - V_* + 4\nu(1-k)}{z - (F^{-1}(V_m/2))' - 2\nu(f_1^{-1}((V_m + V_*(2-k))/(1-k)))'}. \tag{27}$$

Here the primes denote the derivatives with respect to the argument. The initial condition for Eq. (27) has the form

$$z = z_1, \qquad V_m(z_1) = V_{m0}(1-k) - V_*(2-k).$$

Equation (27) is integrated up to $z = z_2$ [z_2 is determined from relation (20): $V_m(z_2) = V_m(z_1)(1-k) - V_*(2-k)$]. We assume that the solution is known in the interval $z_1 < z \leqslant z_2$. We denote it by $V_m = f_2(z)$. In the unloading region, the solution for z from this interval has the form

$$\frac{1}{2}V = \begin{cases} F(\xi + (V/2+\nu)z \\ \quad - (1-k)\nu f_1^{-1}((V + V_*(2-k))/(1-k))), & V_m(z) \geqslant V \geqslant V_m(z_2), \\ F(\xi + (V/2+\nu)z \\ \quad - (1-k)\nu f_2^{-1}((V + V_*(2-k))/(1-k))), & V_m(z_2) > V > V_m(z) - \Delta V(z), \\ F(\xi + (V/2+\nu k)z), & V_m(z) - \Delta V(z) \geqslant V > 0. \end{cases} \tag{28}$$

Similarly, for the nth interval $z_{n-1} < z \leqslant z_n$, where $z_0 = 0$ and z_n is determined from the equation $V_m(z_n) = V_m(z_{n-1})(1-k) - V_*(2-k)$ ($n = 1, 2, \ldots, N$), the solution $V_m(z)$ is found from Eq. (27), in which f_1^{-1} should be replaced by f_{n-1}^{-1} [$V_m = f_{n-1}(z)$ is the known solution from the preceding $(n-1)$th interval]. The solution for the unloading wave is constructed in a similar manner as (28). The length of the sequence N is determined by the wave amplitude: if V_* lies within the Nth interval, the sequence is terminated, and the solution is constructed up to $V_m = V_*$.

We consider the case where a triangular pulse is specified on the boundary $z = 0$, i.e., the function F in (16) has the form

$$F(\xi) = V_{m0}(\xi_{\text{long}} - \xi)/L_0 \quad \text{for} \quad \xi_{\text{long}} - L_0 \leqslant \xi \leqslant \xi_{\text{long}}, \qquad F(\xi) = 0 \quad \text{for} \quad \xi > \xi_{\text{long}}.$$

For $n = 1$, from (23) we obtain

$$z = b(y_0^2/y^2 - 1), \qquad 0 < z \leqslant z_1, \quad z_1 = b(y_0^2/[(1-k)y_0 - \beta]^2 - 1), \tag{29}$$

where $y(z) = V_m(z) - V_* + 4\nu(1-k)$, $y_0 = y(0)$, $b = L_0/V_{m0}$, and $\beta = 2V_* - 4\nu k(1-k)$. The solution for the unloading wave is determined from (24):

$$\frac{1}{2}V = \begin{cases} (\xi_{\text{long}} - \xi - \nu z)/(z+b), & V_m(z) \geqslant V \geqslant V_{m0}(1-k) - V_*(2-k), \\ (\xi_{\text{long}} - \xi - \nu z - \nu(1-k)z_{pe}(V))/(z+b), & V_{m0}(1-k) - V_*(2-k) > V \\ & \qquad > V_m(z)(1-k) - V_*(2-k), \\ (\xi_{\text{long}} - \xi - \nu k z)/(z+b), & V_m(z)(1-k) - V_*(2-k) \geqslant V > 0. \end{cases} \tag{30}$$

Here $z_{pe}(V) = b((1-k)^2 y_0^2/(V - V_* + 4\nu(1-k) + \beta)^2 - 1)$. At the internal boundary points, we have $z_{pe}(V_{m0}(1-k) - V_*(2-k)) = 0$ and $z_{pe}(V_m(z)(1-k) - V_*(2-k)) = z$, i.e., the piecewise solution (30) is continuous for $0 < V < V_m(z)$. However, direct calculations show that the first derivative $\partial V/\partial \xi$ has a discontinuity at these points.

For $n = 2$, we obtain

$$z + b = b\,\frac{y_0^2}{y^2}\Big(1 + 8\nu(1-k)^3\Big(\frac{y+\beta/2}{(y+\beta)^2} - \frac{y_1+\beta/2}{(y_1+\beta)^2}\Big)\Big), \qquad z_1 < z \leqslant z_2. \tag{31}$$

Here $y_1 = (1-k)y_0 - \beta$ and z_2 is determined from (31) if y is replaced by $y_2 = (1-k)y_1 - \beta$. The solution for the unloading wave is found from (28).

Setting $V_* = \nu = 0$ in (29) or (31), we obtain a solution which is well known in hydrodynamics (in a nonlinear-acoustic approximation) [2, 9]. A special feature of attenuation of the amplitude of the plastic shock wave is that it reaches the amplitude of the elastic precursor V_* in a finite time (in hydrodynamics, the shock-wave amplitude tends to zero asymptotically as $z \to \infty$).

REFERENCES

1. A. B. Sawaoka (ed.), *Shock Waves in Materials Science*, Springer-Verlag, New York (1993).
2. O. V. Rudenko and S. I. Soluyan, *Theoretical Fundamentals of Nonlinear Acoustics* [in Russian], Nauka, Moscow (1975).
3. G. Withem, *Linear and Nonlinear Waves*, New York (1974).
4. N. N. Myagkov, "Nonlinear waves in shock-loaded condensed matter," *J. Phys. D, Appl. Phys.*, **27**, 1678–1686 (1994).
5. S. K. Godunov, *Elements of Continuum Mechanics* [in Russian], Nauka, Moscow (1978).
6. D. L. Tonks, "The data shop: A database of weak-shock constitutive data," Report No. LA-12068-MS, Los Alamos (1991).
7. V. N. Kukudzhanov, "Nonlinear waves in elastoplastic media," in: *Wave Dynamics of Machines* [in Russian], Nauka, Moscow (1991), pp. 126–140.
8. V. M. Sadovskii, *Discontinuous Solutions in Dynamic Problems of Elastoplastic Media* [in Russian], Nauka, Moscow (1997).
9. L. D. Landau and E. M. Lifshits, *Course of Theoretical Physics*, Vol. 6: *Fluid Mechanics*, Pergamon Press, Oxford-Elmsford, New York (1987).